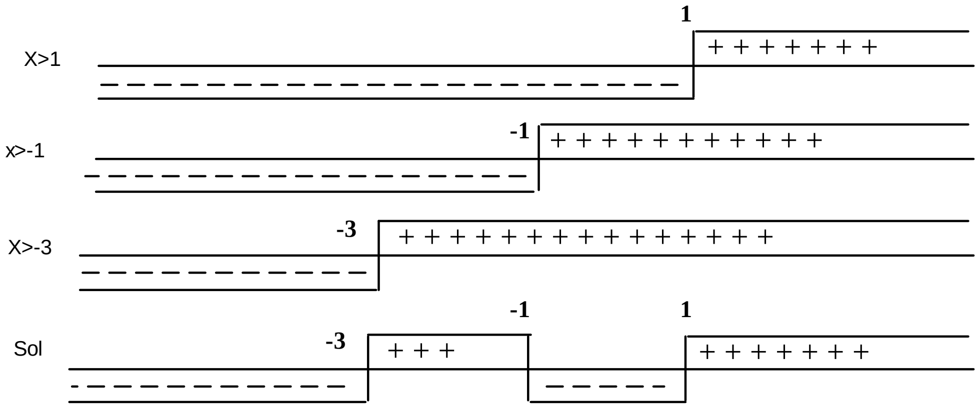
SOLUZIONI ESAME 2005 SPAI Bellinzona

1)

a)

$$x^3+3x^2-x-3\le 0 \rightarrow x^2(x+3)-(x+3)\le 0 \rightarrow (x^2-1)(x+3)\le 0 \rightarrow (x+1)(x-1)(x+3)\le 0$$

$$(x+1)\ge 0 \rightarrow x\ge -1;(x-1)\ge 0 \rightarrow x\ge 1;(x+3)\ge 0 \rightarrow x\ge -3$$

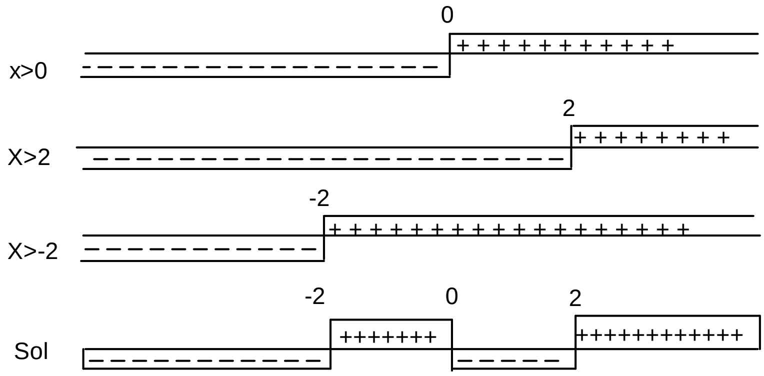


$$x\le -3;\rightarrow -1\le x\le 1$$

b)

$$\frac{2x}{x+2}\ge 2-\frac{9x-2}{2x(x+2)} \rightarrow \frac{2x\cdot 2x+(9x-2)-2\cdot 2x(x+2)}{2x(x+2)}\ge 0 \rightarrow \frac{x-2}{2x(x+2)}\ge 0$$

$$x-2\ge 0 \rightarrow x\ge 2; x>0; x+2>0 \rightarrow x>-2$$



$$-2<x<0 \rightarrow x\ge 2$$

c)

$$\sqrt{7-x}-x+5=0 \rightarrow \sqrt{7-x}=x-5 \rightarrow 7-x=x^2-10x+25 \rightarrow x^2-9x+18=0$$

$$(x-3)(x-6)=0 \rightarrow x_1=3 \text{ inaccettabile} \rightarrow x_2=6$$

$$\text{verifica}: \sqrt{7-3}-3+5\ne 0 \rightarrow \sqrt{7-6}-6+5=0$$

2)
a)

$$\log E = \lambda + \frac{3}{2}R$$

$$\log(2\cdot E_o) = 4{,}4 + \frac{3}{2}R \to 10^{(4{,}4+\frac{3}{2}R)} = 2\cdot E_o \to (4{,}4+\frac{3}{2}R)\log 10 = \log(2) + \log(E_o)$$

$$\log(E_o) = 4{,}4 + \frac{3}{2}R \to 10^{(4{,}4+\frac{3}{2}R)} = E_o \to (4{,}4+\frac{3}{2}R)\log 10 = \log(E_o)$$

$$\log(E_o) - 4{,}4 = \frac{3}{2}R \to R_1 = \frac{2}{3}[\log(E_o) - 4{,}4]$$

$$\log(2) + \log(E_o) - 4{,}4 = \frac{3}{2}R \to R_2 = \frac{2}{3}[\log(E_o) - 4{,}4 + \log(2)] = 0 \to \Delta R = \frac{2}{3}\log(2) \cong 0{,}2$$

b)

$$\log(5{,}62\cdot 10^{17}) = \lambda + \frac{3}{2}8{,}9 \to \lambda \cong 4{,}4$$

c)

$$9^x - 3^{x+1} + 2 = 0 \to (3^x)^2 - 3(3^x) + 2 = 0 \to t = (3^x)$$

$$t^2 - 3t + 2 = 0 \to (t-1)(t-2) = 0$$

$$t_1 = 1; t_2 = 2 \to 3^x = 1 \to x_1 = 0 \to 3^x = 2 \to x_2 = \frac{\log(2)}{\log(3)} \cong 0{,}631$$

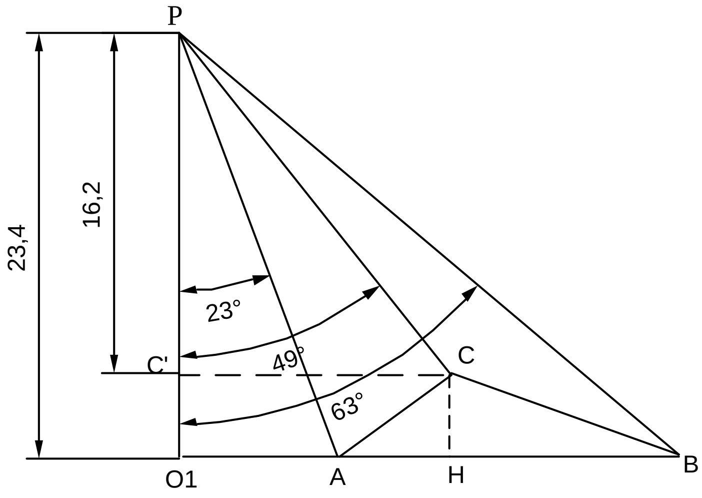
d)

$$\log_2(x) + \log_2(x+4) + 1 = 0 \to C.E: x > 0; x+4 > 0 \to x > -4 \to in\sin tesi\ x > -4$$

$$\log_2(x) + \log_2(x+4) + \log_2(2) = \log(1) \to \log_2[(2x)(x+4)] = \log_2(1)$$

$$2x^2 + 8x - 1 = 0 \to x_{1,2} = \frac{-8 \pm \sqrt{64+8}}{4} \to x_{1,2} = \frac{-4 \pm 3\sqrt{2}}{2} \to x_1 = -4{,}12\ impos s;\ x_2 = 0{,}12$$

3)a)



$$\frac{O_1B}{PO_1} = tg63° \rightarrow O_1B = 23{,}4 \cdot tg63°$$

$$AO_1 = 23{,}4 \cdot tg23° \rightarrow AB = 23{,}4 \cdot tg63° - 23{,}4 \cdot tg23° \cong 35{,}99m$$

$$\frac{CC'}{16{,}2} = tg49° \rightarrow CC' = 16{,}2 \cdot tg49°$$

$$AH = CC' - O_1A = 16{,}2 \cdot tg49° - 23{,}4 \cdot tg23° \cong 8{,}703m$$

$$\frac{CH}{AH} = tg\alpha \rightarrow \alpha = tg^{-1}\left(\frac{7{,}2}{8{,}703}\right) = 39{,}6°$$

$$BH = AB - AH = 35{,}99 - 8.703 = 27{,}29m$$

$$\frac{CH}{BH} = tg\beta \rightarrow \alpha = tg^{-1}\left(\frac{7{,}2}{27{,}29}\right) = 14{,}78°$$

b)

$$2\cos^2(x) - 3\cos x + 1 = 2sen^2(x) \rightarrow 2\cos^2(x) - 2sen^2(x) - 3\cos x + 1 = 0$$

$$2\cos^2(x) - 2[1 - \cos^2(x)] - 3\cos(x) + 1 = 0 \rightarrow 4\cos^2(x) - 3\cos(x) - 1 = 0 \rightarrow t = \cos(x)$$

$$4t^2 - 3t - 1 = 0 \rightarrow t_1 = -\frac{1}{4}; t_2 = 1$$

$$i)\cos(x) = -\frac{1}{4} \rightarrow x_{1,2} = \pm 104.48° + k \cdot 360° = \pm 1.82 + k \cdot 2\pi$$

$$ii)\cos(x) = 1 \rightarrow x_{1,2} = \pm 0 + k \cdot 360° = \pm 0 + k \cdot 2\pi$$

$$Sol(0;360°) : \{0;104{,}48°;255{,}52°;360°\}$$

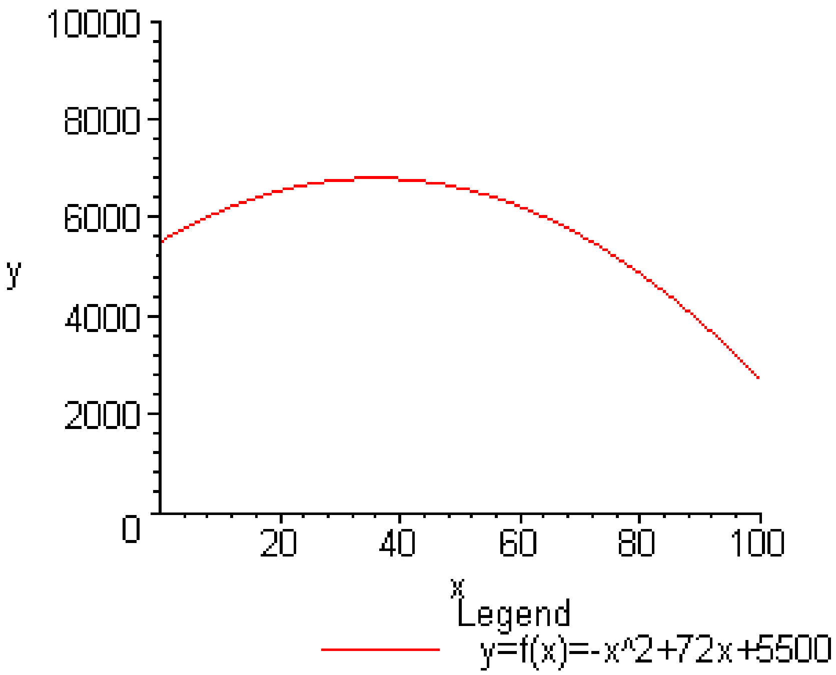
Nome e cognome: ……………………………………………….

4) truffes 70Fr/kg; ovetti 55Fr/kg; pralinés (112-x) Fr/kg

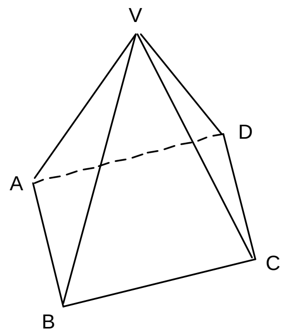
a) $C = 20 \cdot 70 + (112 - 20) \cdot 20 + 60 \cdot 55 = 6540 Fr$

b)
$$6696 = x \cdot 70 + (112 - x) \cdot x + (100 - 2x) \cdot 55$$
$$x^2 - 72x + 1196 = 0 \rightarrow x_1 = 26; \rightarrow x_1 = 46$$
$$(26;26;48) \rightarrow (46;46;8)$$

c)
$$C = x \cdot 70 + (112 - x) \cdot x + (100 - 2x) \cdot 55$$
$$C = -x^2 + 72x + 5500$$
$$x_v = -\frac{b}{2a} = \frac{-72}{-2} = 36 \rightarrow (36Kg \text{ truffes};36Kg \text{ pralinés};28\text{ Kg ovetti.})$$
$$C(36) = -(36)^2 + 72 \cdot 36 + 5500 = 6796\ Fr.$$



Nome e cognome: ……………………………………………….



5)

$$A=\begin{Bmatrix}5\\-3\\9\end{Bmatrix};B=\begin{Bmatrix}-3\\3\\4\end{Bmatrix};C=\begin{Bmatrix}0\\7\\z\end{Bmatrix};V=\begin{Bmatrix}6\\10\\12\end{Bmatrix};$$

$$\vec{VA}=\vec{OA}-\vec{OV}=\begin{Bmatrix}5\\-3\\9\end{Bmatrix}-\begin{Bmatrix}6\\10\\12\end{Bmatrix}=\begin{Bmatrix}-1\\-13\\-3\end{Bmatrix}$$

$$\vec{VB}=\vec{OB}-\vec{OV}=\begin{Bmatrix}-3\\3\\4\end{Bmatrix}-\begin{Bmatrix}6\\10\\12\end{Bmatrix}=\begin{Bmatrix}-9\\-7\\-8\end{Bmatrix}$$

$$\left|\vec{VA}\right|=\sqrt{(-1)^2+(-13)^2+(-3)^2}=\sqrt{179}$$

$$\left|\vec{VB}\right|=\sqrt{(-9)^2+(-7)^2+(-8)^2}=\sqrt{194}$$

$$\cos\alpha=\frac{\vec{VA}\cdot\vec{VB}}{\left|\vec{VA}\right|\cdot\left|\vec{VB}\right|}=\frac{(-1)(-9)+(-13)\cdot(-7)+(-3)(-8)}{\sqrt{179}\cdot\sqrt{194}}=\!\!\rightarrow\alpha=48{,}29°$$

b) c)

$$\vec{VA} x \vec{VB} = \begin{Bmatrix} -1 \\ -13 \\ -3 \end{Bmatrix} x \begin{Bmatrix} -9 \\ -7 \\ -8 \end{Bmatrix} = 83\,\vec{i} + 19\,\vec{j} - 110\,\vec{k}$$

$$\left| \vec{VA} x \vec{VB} \right| = \sqrt{83^2 + 19^2 + (-110)^2} = \sqrt{19350} \cong 139{,}104$$

$$A\Delta ABC = \frac{\sqrt{19350}}{2} \cong 69{,}55$$

$$oppure: A = \frac{1}{2}\left|\vec{VA}\right| \cdot \left|\vec{VB}\right| \cdot sen\alpha = \frac{1}{2} \cdot \sqrt{179} \cdot \sqrt{194} \cdot sen 48{,}286° \cong 69{,}55$$

$$\vec{BA} \cdot \vec{BC} = 0$$

$$\vec{BA} = \begin{Bmatrix} 5 \\ -3 \\ 9 \end{Bmatrix} - \begin{Bmatrix} -3 \\ 3 \\ 4 \end{Bmatrix} = \begin{Bmatrix} 8 \\ -6 \\ 5 \end{Bmatrix} \rightarrow \vec{BC} = \begin{Bmatrix} 0 \\ 7 \\ z \end{Bmatrix} - \begin{Bmatrix} -3 \\ 3 \\ 4 \end{Bmatrix} = \begin{Bmatrix} 3 \\ 4 \\ z-4 \end{Bmatrix}$$

$$\rightarrow 8 \cdot 3 + (-6) \cdot 4 + 5 \cdot (z-4) = 0 \rightarrow z = 4$$

$$\vec{BA} = \vec{CD} = \begin{Bmatrix} 8 \\ -6 \\ 5 \end{Bmatrix} = \begin{Bmatrix} x \\ y \\ z \end{Bmatrix} - \begin{Bmatrix} 0 \\ 7 \\ 4 \end{Bmatrix} \rightarrow x = 8; y = 1; z = 9 \rightarrow D = \begin{Bmatrix} 8 \\ 1 \\ 9 \end{Bmatrix}$$

6) $y = f(x) = \dfrac{4x^2 - 4x - 8}{x^2 - 2x - 8}$

a)b) $x^2 - 2x - 8 = (x-4)(x+2) = 0 \to x_1 = -4 \to x_2 = 2$

$x_3 = -2 \qquad x_4 = 4 \to$ asintoti verticali

dominio: $]-\infty;-2[U]-2;4[U]4;+\infty[$

$\lim f(x)\ se\ x \to \pm\infty = \dfrac{x^2\left(4 - \frac{4}{x} - \frac{8}{x^2}\right)}{x^2\left(1 - \frac{2}{x} - \frac{8}{x^2}\right)} = 4 \to g(x) = 4$ è un asintoto orizzontale

c) zeri: f(x)=0

$\text{f(x)} = 0 \to \text{C.E : x} \neq -2;4$

$\dfrac{4x^2 - 4x - 8}{x^2 - 2x - 8} = 0 \to 4x^2 - 4x - 8 = 0 \to x^2 - x - 2 = 0 \to (x-2)(x+1) = 0$

$x_1 = -1; x_2 = 2$

intersezione asse y : $\text{f}(0) = \dfrac{-8}{-8} = 1$

d) $-\infty;-2;\ positiva \qquad -2;-1;\ negativa \qquad -1;2;\ positiva$

$2;4;\ negativa \qquad 4;+\infty;\ positiva$

x	-4.00	-3.00	-2.00	-1.00	0.00	1.00	2.00	3.00	4.00	5.00	6.00
f(x)	1.50	1.86	as.	0.60	1.00	1.22	1.50	2.20	as.	-0.43	0.25